

Concept Question 3-5: Explain the similarities and differences between the time-shift and frequency-shift properties of the Laplace transform.

A shift by T in the time domain becomes a multiplication by e^{-Ts} in the frequency domain, and a shift by \mathbf{a} in the frequency domain becomes a multiplication by $e^{-\mathbf{a}t}$ in the frequency domain.

$$x(t - T) u(t - T) \longleftrightarrow e^{-Ts} \mathbf{X}(s), \quad T \geq 0. \quad (3.16)$$

(time-shift property)

$$e^{-\mathbf{a}t} x(t) \longleftrightarrow \mathbf{X}(s + \mathbf{a}). \quad (3.20)$$

(frequency shift property)