

Concept Question 2-9: What is the area property of convolution?

The area of the convolution of two signals is the product of the areas of the two signals.

$$\begin{aligned}\text{Area of } y(t) &= \int_{-\infty}^{\infty} y(t) dt \\ &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau \right] dt \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\tau) x(t - \tau) dt d\tau \\ &= \int_{-\infty}^{\infty} h(\tau) \left[\int_{-\infty}^{\infty} x(t - \tau) dt \right] d\tau \\ &= \left[\int_{-\infty}^{\infty} h(\tau) d\tau \right] \left[\int_{-\infty}^{\infty} x(t - \tau) dt \right] \\ &= \text{area of } h(t) \times \text{area of } x(t). \quad (2.76)\end{aligned}$$