

Concept Question 10-8: Why is Tikhonov regularization needed?

Convolution can be implemented using zero-padded 2-D DFTs as $\mathbf{Y}[k_1, k_2] = \mathbf{H}[k_1, k_2] \mathbf{X}[k_1, k_2]$, as given by (Eq. 10.40). So it seems that deconvolution can be implemented using $\mathbf{X}[k_1, k_2] = \mathbf{Y}[k_1, k_2] / \mathbf{H}[k_1, k_2]$, as given by (Eq. 10.41).

But if $\mathbf{H}[k_1, k_2]$ is small, dividing by $\mathbf{H}[k_1, k_2]$ amplifies any noise in $\mathbf{Y}[k_1, k_2]$. So instead of using Eq. (10.41), we can use Tikhonov regularization, which minimizes the Tikhonov criterion given by Eq. (10.45):

$$e = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} [(y[m, n] - h[m, n] * \hat{x}[m, n])^2 + (\lambda \hat{x}[m, n])^2], \quad (10.45)$$

which is accomplished by the Wiener filter of Eq. (10.47).